

Vermont State Mathematics Coalition Talent Search -- November 2017

Test 2 of the 2017-2018 school year

PRINT NAME: _____ Signature: _____

Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School _____ Grade _____

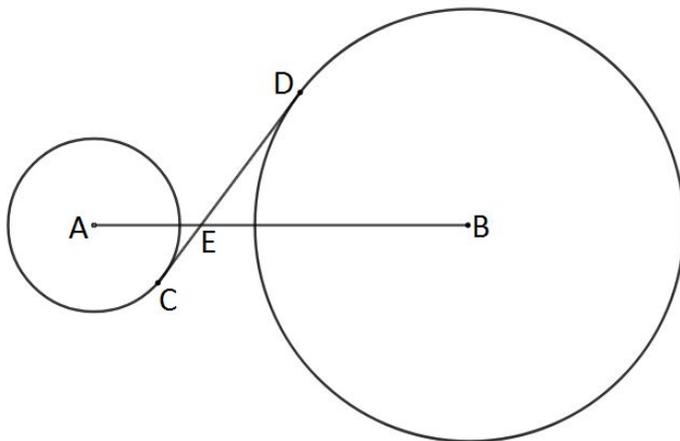
Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@fnwsu.org or be postmarked by **December 18, 2017** and submitted to

Kiran MacCormick
Missisquoi Valley Union High School
175 Thunderbird Drive
Swanton, VT 05488

To receive the next tests via email, clearly print your email address below:

1. Two circles with centers A and B have radii of 8 inches and 20 inches, respectively. The centers are 35 inches apart. If CD is a common internal tangent to the two circles and CD intersects AB at E, what is the length of segment AE?



Answer: _____

2. Find the number of perfect cubes in the set $\{1^1, 2^2, 3^3, 4^4, 5^5, \dots, 2017^{2017}\}$

Answer: _____

3. Find all real solutions to the equation $2 \log(x+16) = 2 + \log(x)$.

Answer: _____

4. A 2017-dimensional hypercube of side length 1 is drawn along with all of its diagonals, yielding a total of $2^{4033} - 2^{2016}$ segments. Compute the median length of these segments.

Answer: _____

5. Prove that $f(n) = 1 - n$ is the only integer-valued function defined on the integers that satisfies the following conditions.

- (i) $f(f(n))=n$, for all integers n ;
- (ii) $f(f(n+2)+2)=n$ for all integers n ;
- (iii) $f(0) = 1$.

Note: For this problem, please include your proof on separate sheets of paper.

6. A set of 15 (distinct) positive integers has the property that the arithmetic mean of the elements in any nonempty subset is an integer. Find the smallest possible value for the largest of the integers.

Answer: _____