

Vermont State Mathematics Coalition Talent Search -- October 2017

Test 1 of the 2017-2018 school year

PRINT NAME: _____ Signature: _____

Note: Your signature indicates that answers provided herein are your own work and you have not asked for or received aid in completing this Test.

School _____ Grade _____

Current Mathematics Teacher: _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final. Your solutions may be e-mailed to kmaccormick@fnwsu.org or be postmarked by **November 3, 2017** and submitted to

Kiran MacCormick
Missisquoi Valley Union High School
175 Thunderbird Drive
Swanton, VT 05488

To receive the next tests via email, clearly print your email address below:

1. Let V be a square and T be an equilateral triangle, and suppose V and T have the same perimeter. What is the ratio of their areas $\frac{\text{area}(V)}{\text{area}(T)}$?

Answer: _____

2. The outside surface of a large cubical box is painted and then broken into unit cubes. If exactly 3174 of the unit cubes were painted on only one side, what is the side length of the large cube?

Answer: _____

3. In how many ways can 4035 be written as the sum of two or more consecutive positive integers in increasing order? For example, $2017 + 2018$ would count, but $2018 + 2017$ would not.

Answer: _____

4. If a and b are positive numbers and $a + b = ab = a^2 - b^2$, then what is a ?

Answer: _____

5. How many ways are there to give 18 indistinguishable candy bars to Anisha, Boris, Chang, Diego, and Elsa in such a way that everyone gets at least one, and no one gets more than 6?

Answer: _____

6. Suppose $p(x) = a_8x^8 + a_7x^7 + \dots + a_2x^2 + a_1x + a_0$ is a polynomial with integer coefficients where $a_8 \neq 0$. Prove that the maximum number of integers n that can satisfy $[p(n)]^2 = 25$ is 8.

Note: For this problem, please include your proof on separate sheets of paper.