

**Vermont State Mathematics Coalition Talent Search** November 2, 2015

Test 2 of the 2015 – 2016 school year

PRINT NAME: \_\_\_\_\_ Signature: \_\_\_\_\_

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School \_\_\_\_\_ Grade \_\_\_\_\_

Current Mathematics Teacher: \_\_\_\_\_

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. **On separate sheets**, in an organized way, show how you solved the problems. For problems that require a proof (indicated after the problem), you will be awarded full credit for a correct proof that is mathematically rigorous with no logical gaps. For problems that require a numerical answer, you will be awarded full credit for a complete correct answer with adequately supported reasoning. Partial credit will be given for correct answers having insufficient justification, numerical approximations of exact answers, incorrect answers with substantially correct reasoning, incomplete solutions or proofs, or proofs with logical errors. For solutions relying on computer assistance, all such computations must be clearly indicated and justified as correct. The decisions of the graders are final." Your solutions may be emailed to [joholson@sbschools.net](mailto:joholson@sbschools.net) or be postmarked by **November 30, 2015** and submitted to:

Jean Ohlson  
Vermont State Math Coalition  
PO Box 384  
Charlotte, VT 05445

**To receive the next tests via email, clearly print your email address below:**

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1) Observe that 25 is a perfect square, and the integer obtained by increasing all of its digits by 1 (namely, 36) is also a perfect square. Find the next smallest integer possessing this property: namely, that it is a perfect square, and the integer obtained by increasing all of its digits by 1 is also a perfect square. *Note: it is not possible to increase by 1 the digits of any number containing a digit 9, because  $9+1$  is not a base-10 digit.*

Answer: \_\_\_\_\_

2) In circle  $O$ , chords  $AB$  and  $CD$  intersect at  $E$ . If  $AE = 3 \cdot BE$  and  $CE = 12 \cdot DE$ , find  $\frac{AC}{BD} + \frac{BC}{AD}$ .

Answer: \_\_\_\_\_

3) A random function  $f$  from the set  $S = \{1,2,3,4\}$  to the set  $T = \{5,6,7\}$  is chosen, and a random function  $g$  from  $T$  to the set  $U = \{8,9\}$  is chosen. Find the probability that the range of the composite function  $g \circ f$  is  $U$ .

Answer: \_\_\_\_\_

4) In rectangle  $ABCD$ , there exist points  $E$  on  $BC$  and  $F$  on  $CD$  such that triangle  $AEF$  is equilateral. If the area of  $AEF$  is  $16\sqrt{3}$  and the area of  $ABCD$  is  $34\sqrt{3}$ , find the area of triangle  $CEF$ .

Answer: \_\_\_\_\_

5) An "odd Egyptian fraction decomposition into  $n$  terms" of a rational number  $\frac{p}{q}$  is a representation  $\frac{p}{q} = \frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}$  where  $a_1 < a_2 < \dots < a_n$  are odd positive integers. For example,  $\frac{3}{5} = \frac{1}{3} + \frac{1}{5} + \frac{1}{15}$  is an odd Egyptian fraction decomposition of  $\frac{3}{5}$  into three terms.

- Prove that there is no odd Egyptian fraction decomposition of 1 into 2016 terms.
- Prove that there is an odd Egyptian fraction decomposition of 1 into 2015 terms.

*Note: For this problem, please include your proof on a separate sheet of paper.*

6) Let  $a$  and  $b$  be real numbers such that  $a^4b^3 + a^3b^4 = 2160$  and  $a^5b^2 + a^2b^5 = 29520$ . Find the value of  $(a + 7)(b + 7)$ .

Answer: \_\_\_\_\_