

Vermont State Mathematics Coalition Talent Search February 10, 2014
Test 4 of the 2013 – 2014 school year

PRINT NAME: _____ Signature: _____

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School _____ Grade _____

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions may be emailed to joholson@sbschools.net or be postmarked by March 10, 2014 and submitted to:

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To receive the next tests via email, clearly print your email address below:

Problem 1.

In triangle ABC , $AC = 7$. D lies on AB such that $AD = BD = CD = 5$. Find BC .

Answer: _____

Problem 2.

The increasing sequence $S = \{2, 3, 5, 6, 7, 10, 11, \dots\}$ consists of all positive integers which are neither a perfect square nor a perfect cube. What is the 2014th term of S ?

Answer: _____

Problem 3.

Let $N = 6 + 66 + 666 + \dots + \underbrace{666 \dots 6}_{2014 \text{ 6's}}$ where the last number has 2014 sixes. What is the sum of the digits of $27N$?

Answer: _____

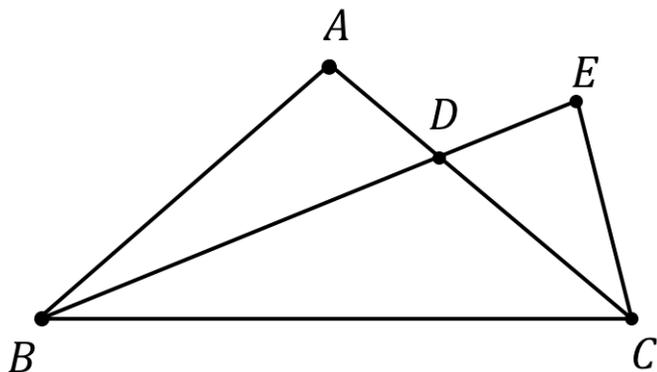
Problem 4.

Find all real x for which $1 + \sqrt{x+1} + \sqrt{2x+1} = \sqrt{7x+1}$.

Answer: _____

Problem 5.

In isosceles triangle ABC the base angles at B and C are 40° . The bisector of angle B meets AC at D and BD is extended to E so that $DE = AD$ (see figure below). How big is $\angle E$?



Answer: _____

Problem 6.

Find the smallest positive integer k such that k^3 ends in the digits 11111 (in base 10).

Answer: _____

Problem 7.

Let $S = \{a, b, c, d, e, f\}$. Find the value of

$$\sum_{A \subseteq S} \sum_{B \subseteq S} |A \cap B|$$

Note: $A \cap B$ Indicates the intersection of sets A and B and $A \subseteq S$ indicates A is a subset of S that is possibly equal to S , and $|H|$ denotes the number of elements in the set H .

Answer: _____

Problem 8.

Find the number of complex numbers z ($\neq 0, 1$) such that z, z^2 , and z^4 form an equilateral triangle in the complex plane.

Answer: _____