Vermont State Mathematics Coalition Talent Search February 5, 2013
Test 3 of the 2012 – 2013 school year

PRINT NAME:_________________________ Signature:_____________________

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade____________

Directions: Solve as many of the problems as you can and list your answers on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions may be emailed to johlson@sbschools.net or be postmarked by March 5, 2013 and submitted to:

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Vermont State Math Coalition
PO Box 384
Charlotte, VT 05445

To receive the next tests via email, clearly print your email address below:

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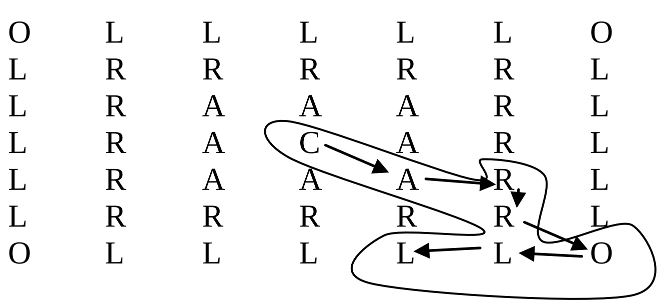
Problem 1.

Given the array of letters shown, find the number of ways that the sequence C-A-R-R-O-L-L can be obtained if consecutive letters in the word are a single horizontal, vertical, or diagonal move away from each other.



An example is shown on the grid.

Answer: ____________

The Vermont Math Coalition's Talent Search test is prepared by Jean Ohlson, Robert Poodiack and Evan Dummit, a graduate mathematics student at the University of Wisconsin, Madison WI. With additional support from Tony Trono a retired math teacher from Burlington High School.

Problem 2.

Three non-overlapping semicircles of radius 1 are contained in a $1x\ d$ rectangle. Find the smallest possible value for d.

Answer: ____________

Problem 3.

How many different 3 x 3 arrays of non-negative integers are able to be constructed such that each of the three horizontal sums and each of the three vertical sums is equal to 7, and the sums on the two major diagonals are 9 and 10.

Answer: ____________

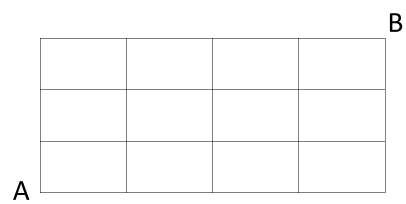
Problem 4.

If the parabola $y = x^2 - c$ intersects the circle $x^2 + y^2 = c^2$in three distinct points forming an equilateral triangle, find all possible values of c.

Answer: ____________

Problem 5.

The figure below shows a street plan of twelve square blocks. A person P goes from point A to point B, and a second person Q goes from B to A. Both of them (P and Q) leave at the same time with the same speed, following shortest paths on the grid. At each corner they choose among the possible streets with equal probability. What is the probability that P meets Q?



Answer: ____________

Problem 6.

For a positive number such as 3.14, we call 3 the *integer part*, and 0.14 the *fractional part*. Find a positive number such that the fractional part, the integer part, and the number itself are three consecutive terms

a. In an arithmetic sequence
b. In a geometric sequence.

Answer: a) ____________

b) ____________

Problem 7.

Suppose we roll $N \geq 3$ standard 6-sided dice. What value(s) of N will maximize the probability of obtaining exactly three threes?

Answer: ____________

Problem 8.

Find all x, $0 \leq x \leq \pi$, for which $\cos(x) \cdot \cos(2x) \cdot \cos(4x) = \frac{1}{8}$.

Answer: ____________