Vertmont State Mathematics Coalition Talent Search November 10, 2008

Test 2 of the 2008 – 2009 school year

PRINT NAME:_________________________ Signature:_____________________

Note: Your signature indicates that answers provided herein is your own work and you have not asked for or received aid in completing this Test.

School____ Grade____________

Directions: Solve as many of the problems as you can and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for inadequately supported correct answers and/or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. Solutions that display creativity, ingenuity and clarity may receive special recognition and commendation. Your solutions must be postmarked by December 1, 2008 and submitted to:

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1. $(x, y, z) = (4, 3, 9)$ is an ordered triple of real numbers which satisfies the following system of equations.

$$z^x = y^{2x}, \quad 2^z = (2)(4^x) \text{ and } x + y + z = 16$$

Find another ordered triple (x, y, z) of real numbers satisfying the same system.

2. Consider the 2 × 3 rectangle with vertices at (0, 0), (2, 0), (0, 3) and (2, 3). The rectangle is rotated 90° clockwise around the point (2, 0), so that the vertex originally at (0, 3) ends up at (5, 2). Now rotate 90° clockwise around the point (5, 0), then 90° clockwise around the point (7, 0), and finally 90° clockwise around the point (10, 0). (The side originally on the x-axis is now back on the axis.) Find the area of the region above the x-axis and below the curve traced out by the point that was initially at (1,1)

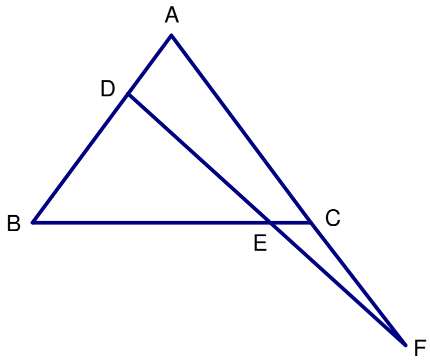
Answer:____________________

3. $(\phi - 1)\sqrt{a\phi + b} = \sqrt{\phi + 2}$ where $\boldsymbol{a}$ and $\boldsymbol{b}$ are integers and ϕ, the Golden Ratio, is $\dfrac{1+\sqrt{5}}{2}$.

If $21a^3 + 24b^3 + K = 2008$, find $\boldsymbol{K}$.

Answer:____________________

4. In ΔABC, AB = AC =208, AD = 52 and CF = 156. The ratio of the area of ΔCEF to ΔBDE can be written as $\frac{m}{n}$. Compute the ordered pair (m,n)



Answer:____________________

5. The radius of a circle is 10. The area of a sector of the circle (central angle **θ** is in radians) is numerically equal to twice the perimeter of the sector. Find angle **θ.**

Answer:____________________

6. Six boxes are numbered 1 through 6. How many ways are there to place 20 identical marbles into these boxes so that none of them are empty?

Answer:____________________

7. A pan of length 24 cm, width 19 cm and height 15 cm is filled with water to a depth of 3 cm. Lead cubes of edge 4 cm are placed flat on the bottom of the pan. When the $\boldsymbol{n}^{\text{th}}$ cube is placed in the pan, all cubes are covered with water for the first time. Compute $\boldsymbol{n}$.

Answer:____________________

8. Find the number of distinct elements in the set $S = \left\{ floor\left(\frac{x^2}{2009} \right) \right\}$ as x runs over the set {1, 2, ... , 2009}.

Note: The *floor* function (sometimes called the greatest integer function) of a real number *x* is a function that returns the largest integer less than or equal to *x*.

Answer:____________________

´ The Math Coalition is grateful for problem contributors for this test including Middlebury College professors Michael Olinick, Bill Peterson Peter Schumer and Frank Swenton. Also contributing is Toni Trono, retired Burlington High School math teacher and Evan Dummit.