VERMONT STATE MATHEMATICS COALITION TALENT SEARCH January 9, 2007

Test 3 of the 2006 - 2007 school year (Test 4 arrives at schools February 20, 2007)

Student Name _________________________ School ______________________________

Grade ________ Math Department Head _________________________________

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by February 6, 2007 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. For Coalition information and a copy of the test:
http://www.vermontinstitutes.org/vsmc/talent-search/

1. At the stroke of midnight on New Year's Eve 2006-2007, Amy had savings of $135 stored away in her sock drawer. Starting on New Year's Day and for every day in the year, Amy will put $1 into her savings. On every fifth day (first time: January 5) Amy will also add into her savings the $5 she receives for kitchen chores. On every seventh day (first time: January 7) Amy will withdraw $7 from her savings to pay her entry fee in the Really Very Hard Problems Math Contest. How much money will Amy have in her savings at the stroke of midnight on New Year's Eve 2007-2008?

Answer: _______________

2. Al is 17 years older than Barbara. If his age is written after her age, the result is a four digit perfect square number. Thirteen years from now, the previous statement is again true. Find Barbara's present age.

Answer: _______________

3. An arithmetic progression has been started with the first three terms: *a*, *b*, *c*. The third term is four times as large as the first term. When the third term is replaced by its reciprocal, then these three terms form a geometric progression. Find the smallest possible value of the fourth term of the geometric progression.

Answer: _______________

4. Both z_1 and z_2 are non-real roots of the equation $x^4 - 3x^3 + 5x^2 - 27x - 36 = 0$.
Evaluate $(z_1)^4 + (z_2)^4$.

Answer: _______________

5. For the following system of equations

$$\begin{aligned} (c+1)x + 4y - 2z &= 9 \\ -2x - 3y + 2cz &= -10 \\ -4x + cy + 3z &= 17 \\ x + y + z &= 0 \end{aligned}$$

a) Find the integral solution for c.
b) Solve the system for (x, y, z).

Answer: a) __________ Answer: b) _______________

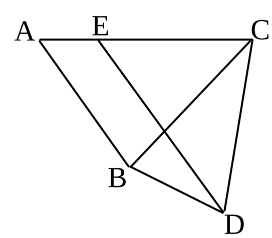
6. For all ordered pairs of positive integers (x, y), $f(x, y)$ is defined by

1) $f(x,1) = x$
2) $f(x, y) = 0$ if $y > x$
3) $f(x+1, y) = y[f(x, y) + f(x, y-1)]$

For what value of n is $f(n, n) = 5040$.

Answer: _______________

7. In ΔABC, E lies on AC with AE = 3 and CE = 8. In ΔBCD, BC = CD = 9. Given that DE = 11 and AB = 8, find the ratio of the area of ΔBCD to the area of quadrilateral ABDE.



Answer: _______________

8. Write an equation in a, b, c, and d by eliminating x, y, and z from the following set of equations.

$\cos(x) + \cos(y) + \cos(z) = a$
$\sin(x) + \sin(y) + \sin(z) = b$
$\cos(2x) + \cos(2y) + \cos(2z) = c$
$\sin(2x) + \sin(2y) + \sin(2z) = d$

Answer: _______________