VERMONT STATE MATHEMATICS COALITION TALENT SEARCH November 15, 2005

Test 2 of the 2005 - 2006 school year (Test 3 arrives at schools January 3, 2006)

Student Name ________________________ School ____________________________

Grade ________ Math Department Head _______________________________

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by December 13, 2005 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. (For Coalition information and a copy of the test: http://www.state.vt.us/educ/vsmc)

1. A family consists of Bob and Marge and their parents, Janet and Steven. Marge is 3 years older than Bob, and Janet is 4 years older than Steven. The total of their ages is now 79 years. Six years ago, the sum of the ages of the family members was 56 years. How many more years will it take for Bob to be 16 years old?

Answer: ______________________

2. Find all integral values of x for which the polynomial P(x) = $x^4 + 4x^3 + 6x^2 + 4x + 5$ will be a prime number.

Answer: ______________________

3. Three consecutive integers have a product of the form abcabc, where a, b, and c are all different digits. Find a, b, and c.

Answer: ______________________

4. A "perfect triangle" is a triangle with integral sides for which the perimeter is numerically equal to the area. Let T_n (a, b, c) where n = 1, 2, 3, 4, 5, 6, 7 be a triangle with sides of lengths a, b, and c. These will be the seven triangles: T_1(6, 8, 10), T_2(4, 11, 14), T_3(5, 12, 13), T_4(9, 10, 17), T_5(7, 15, 20) , T_6(6, 25, 29), and T_7(18, 21, 23).

a) Which of the seven triangles are perfect triangles?

b) Is it possible to put two of the given triangles together without overlapping to form one larger triangle? If so, which can be placed together?

Answer: a) _____________________ b) ______________________

5. In the given nine by nine Latin Square, each of the digits 1 through 9 is used in every row and in every column. The nine by nine Latin Square is composed of nine blocks with each having 3 rows and 3 colums. Each of these arrays contains the digits 1 through 9.

Given the numbers that have been filled in (for example N(4, 6) = 9), fill in the other numbers. If it helps you to get started, N(9, 7) = 4.

7			**1**					**2**
					6		**8**	
			8			**1**		**9**
		7			**9**		**1**	
	9	**3**				**5**	**4**	
	6		**4**			**9**		
3		**8**			**4**			
	4		**3**					
1					**5**			**3**

Note: This is a sudoku (or suudoku) puzzle, with these puzzles first appearing in Japan in about 1988. “Suudoku” contains syllables from a Japanese sentence that can be translated as “The number is limited to a bachelor”, with the understanding that a number in a row or column or block is single—not paired with one like it. Information of this note supplied by Dr. Satoshi Takahashi.

6. Triangle ABC is inscribed in a circle. The bisector of angle B intersects AC at point D and intersects the circle at point E. Given the lengths AB = 35, BC = BD = 22, find the length of DE.

Answer: ______________________

7. For the triangle with angles A, B, and C, the following trig equation holds;
$\sin^2(B) + \sin^2(C) - \sin^2(A) = \sin(B) \cdot \sin(C)$. Find the number of degrees in angle A.

Answer: ______________________

8. The complex number $4 - i\sqrt{5}$ is a root of the equation $(x-3)^4 + (x+k)^4 + 8 = 0$, where k is an integer. a) Find the other roots of the equation. b) Find k.

Answer: a) ________________________ b) ________________