

Test 4 of the 2004 - 2005 school year

(This test completes testing for 2004-2005)

Student Name _____

School _____

Grade _____

Math Department Head _____

Directions: Solve as many as you can of the problems and list your solutions on this sheet of paper. On separate sheets, in an organized way, show how you solved the problems. You will be awarded full credit for a complete correct answer which is adequately supported by mathematical reasoning. You can receive half credit for correct answers which are the result of guesses, conjectures or incomplete solutions. Included as incomplete solutions are solutions that list some, but not all, solutions when the problem asks for solutions of equations. The decisions of the graders are final. You may earn bonus points for "commendable solutions"- solutions that display creativity, ingenuity and clarity. Your answers and solutions must be postmarked by March 15, 2005 and submitted to Tony Trono, Vermont State Mathematics Coalition, 419 Colchester Avenue, Burlington, VT 05401. (For Coalition information and a copy of the test: <http://www.state.vt.us/educ/vsmc>)

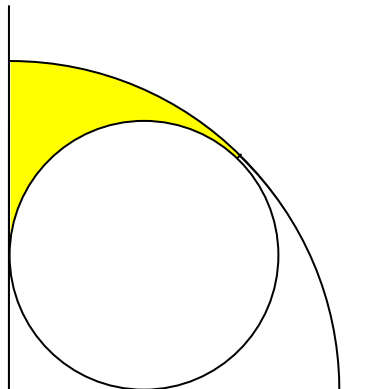
1. Let $f(n) = 3n^3 + 7n^2$ for integers n . Find the sum of the smallest ten integers n for which $f(n)$ is a perfect square.

Answer: _____

2. You are given the simultaneous equations $2x + 3y + 4z = a$, $x + 3y + 5z = b$, and $3x + y + z = c$. Express $x + y + z$ in terms of a , b , and c .

Answer: _____

3. A circle of radius 2 is inscribed in the pictured quadrant of a circle. Find the area of the shaded section.



Answer: _____

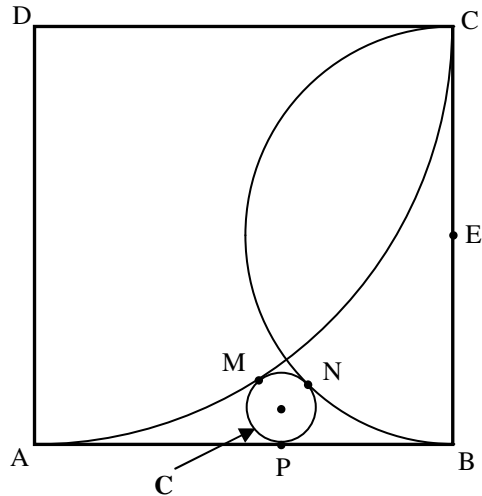
4. Solve the cryptarithm $p^r + q^r + r^r = pqr$, where each of $p, q,$ and r is a distinct decimal digit.

Answer: $p = \underline{\hspace{1cm}}, \quad q = \underline{\hspace{1cm}}, \quad r = \underline{\hspace{1cm}}$

5. Determine the smallest integer k for which the roots of the equation $x^2 + 3(k-2)x + 8k - 6 = 0$ are unequal negative numbers.

Answer: $\underline{\hspace{2cm}}$

6. ABCD is a square of side 4. D is the center of a circle of radius 4. Point E, the midpoint of CB, is at the center of a circle of radius 2. Circle C is tangent at M, N and P to the two circles and to the base of the square. Find the radius, in exact form, of circle C.



Answer: $\underline{\hspace{2cm}}$

7. The polynomial function $f(x)$ has degree 5, $f(x)$ is divisible by x^3 , and $f(x) - 1$ is divisible by $(x-1)^3$. If $f(2) = 2^n$, find n .

Answer: $\underline{\hspace{2cm}}$

8. The lengths of the sides of a triangle are 7, 8, and 9. Point P lies on the side of length 9, and point Q lies on the side of length 8. The line segment PQ cuts the area of the triangle in half and also cuts the perimeter in half. Find the length of PQ.

Answer: $\underline{\hspace{2cm}}$